EE421/521 Image Processing

Lecture 12a
IMAGE RECONSTRUCTION

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Introduction

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Image Recovery

- Image Restoration
  - Undoing imaging degradations
- Image Reconstruction
  - Filling in for missing data

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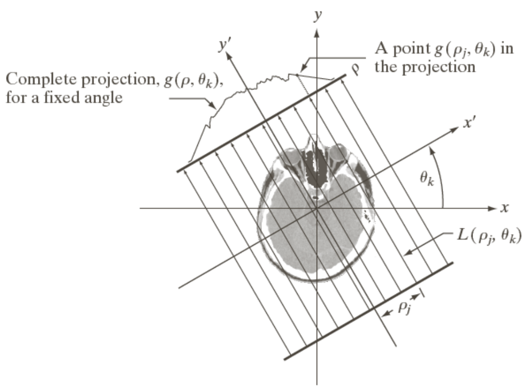
Reconstruction from CT Projections



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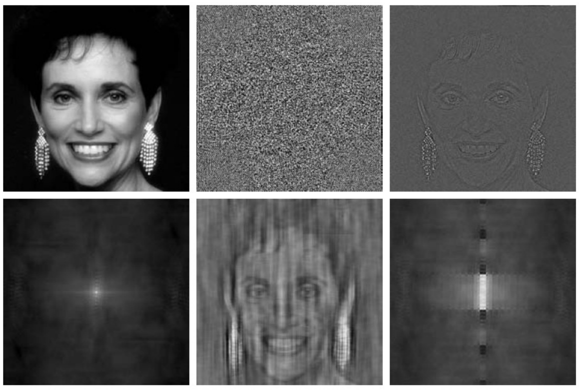
Reconstruction from Fourier Phase

Reconstruction from Magnitude-Only and Phase-Only Images

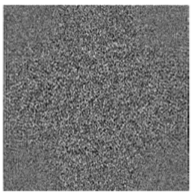
Fig. 4.24(a). (f)



a b c
d e f

FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

Reconstruction from Phase-Only Image



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Finite Image Support Assumption

Image with finite support

Observed Fourier phase

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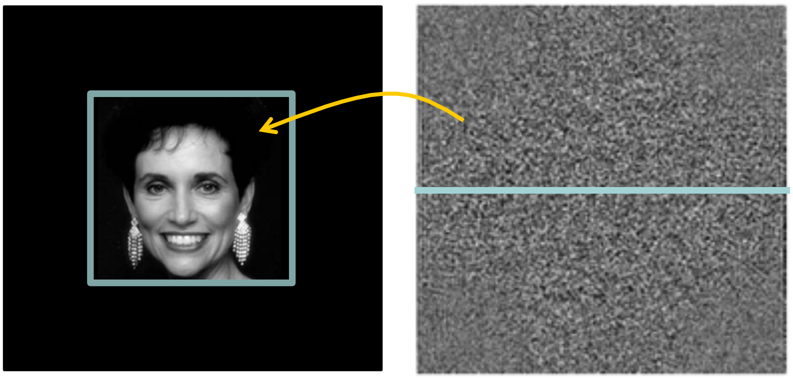
Why Reconstruction is Possible



Unknown intensities in less than **half** of the image

Unique information in **half** of the Fourier phase

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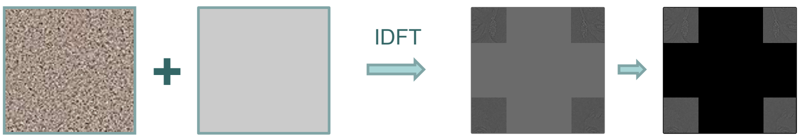
Reconstruction Algorithm: Initialization



Observed Fourier phase

Start with uniform Fourier magnitude

Resulting image after IDFT

Image with finite support constraint

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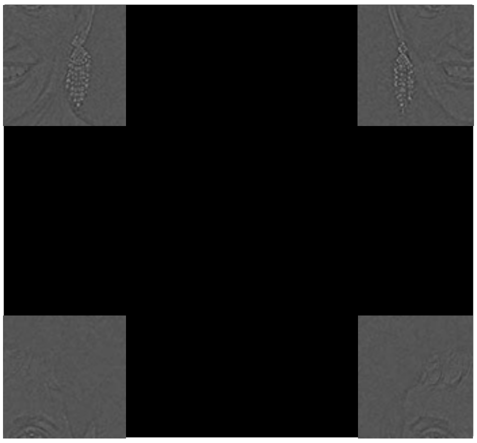
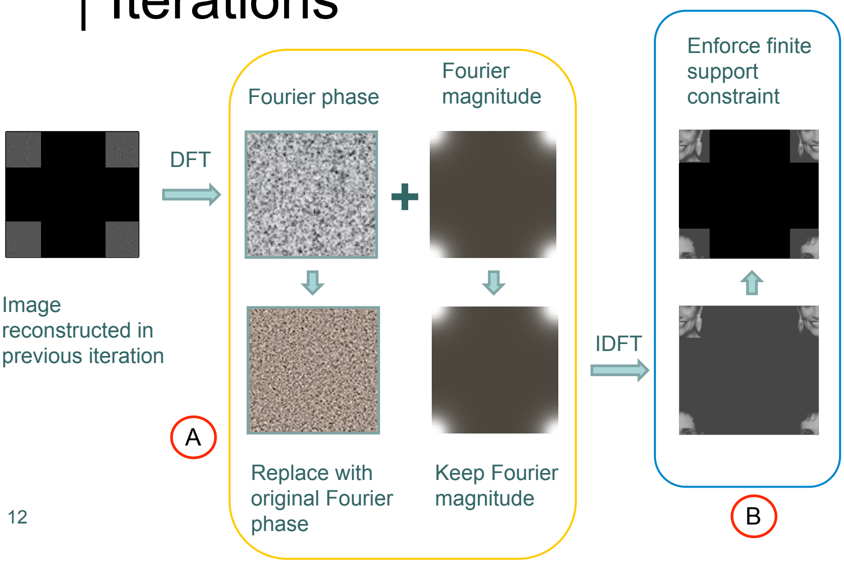
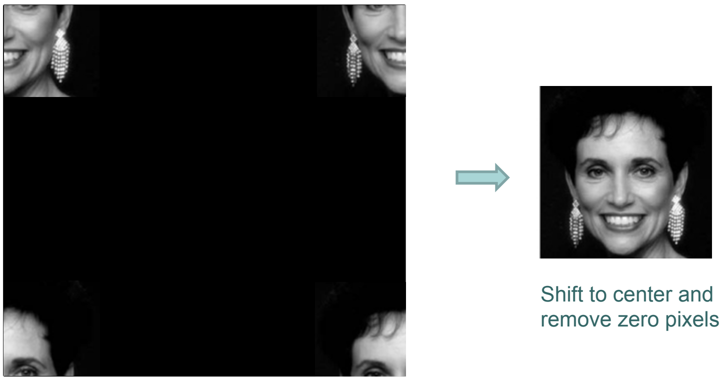
Finite Support Constraint



Reconstruction Algorithm: Iterations



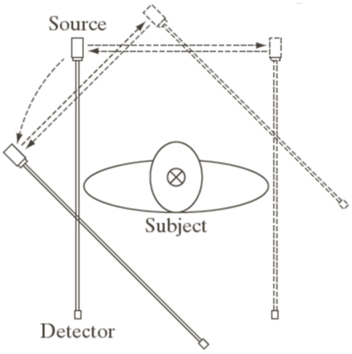
After Several Iterations of A & B…

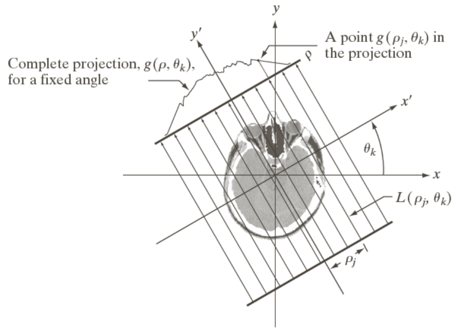


Shift to center and remove zero pixels

Reconstruction from CT Projections

Computed Tomography





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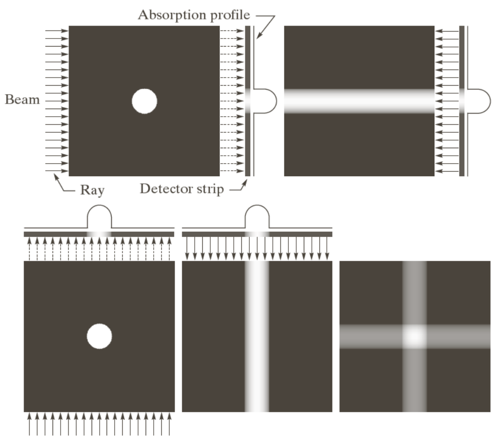
Can We Recover the Image from its Projections?



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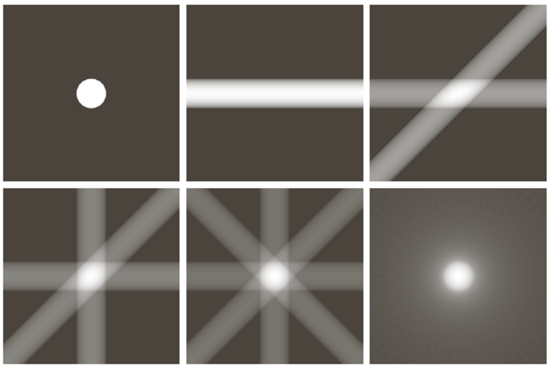
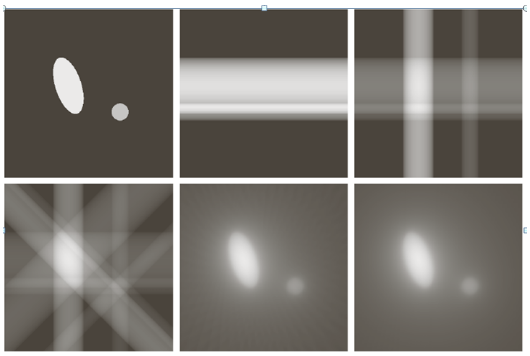
Reconstruction via Backprojections



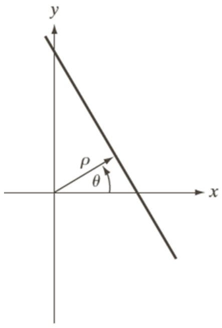
Blurring Caused by Backprojections



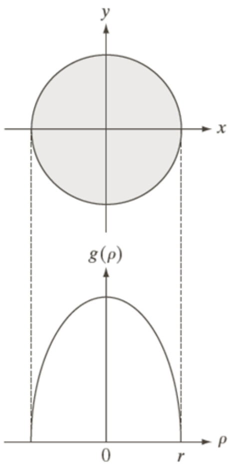
a b c
d e f

FIGURE 5.34 (a) A region with two objects. (b)–(d) Reconstruction using 1, 2, and 4 backprojections 45° apart. (e) Reconstruction with 32 backprojections 5.625° apart. (f) Reconstruction with 64 backprojections 2.8125° apart.

Projection of a Disk



Same in every direction



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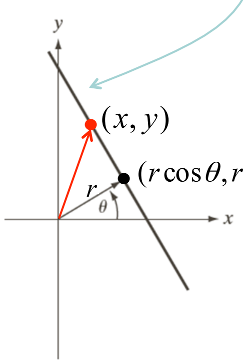
Series of Projections: Radon Transform

$$g(r,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - r)\,dx\,dy$$

Integration along a line



$$(x,y)\bullet(r\cos\theta, r\sin\theta) = r^2$$

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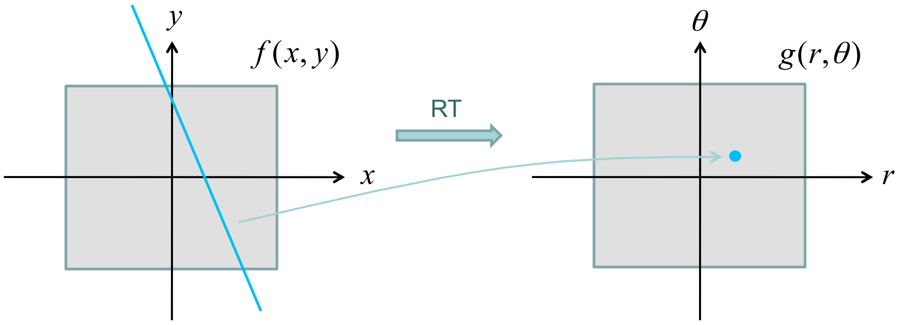
Radon Transform (Sinogram)



How can we get back $f(x,y)$ from $g(r,\theta)$?

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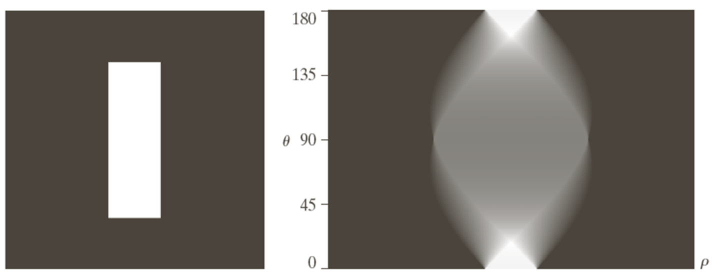
Radon Transform Example



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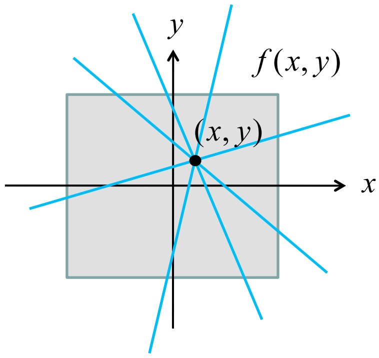
Reconstruction with Backprojections

$$\hat{f}(x,y) = \int_{-\pi/2}^{\pi/2} \int_{-\infty}^{\infty} g(r,\theta)\, \delta(x\cos\theta + y\sin\theta - r)\, dr\, d\theta$$



Reconstruction with Backprojections

$$\hat{f}(x,y) = \int_{-\pi/2}^{\pi/2} \int_{-\infty}^{\infty} g(r,\theta)\, \delta(x\cos\theta + y\sin\theta - r)\, dr\, d\theta$$

$$\hat{f}(x,y) = \int_{-\pi/2}^{\pi/2} g(x\cos\theta + y\sin\theta, \theta)\, d\theta$$

$$\hat{f}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y') \int_{-\pi/2}^{\pi/2} \delta\big((x'-x)\cos\theta + (y'-y)\sin\theta\big)\, d\theta\, dx\, dy$$

Reconstruction with Backprojections

$$\hat{f}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y') \int_{-\pi/2}^{\pi/2} \delta\big((x'-x)\cos\theta + (y'-y)\sin\theta\big)\, d\theta\, dx\, dy$$

$$\hat{f}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\big((x-x')^2 + (y-y')^2\big)^{-1/2}\, dx'\, dy'$$

$$\hat{f}(x,y) = f(x,y) ** \frac{1}{\sqrt{x^2+y^2}}$$

Causes blurring

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Approach 1: Fourier Slice Theorem

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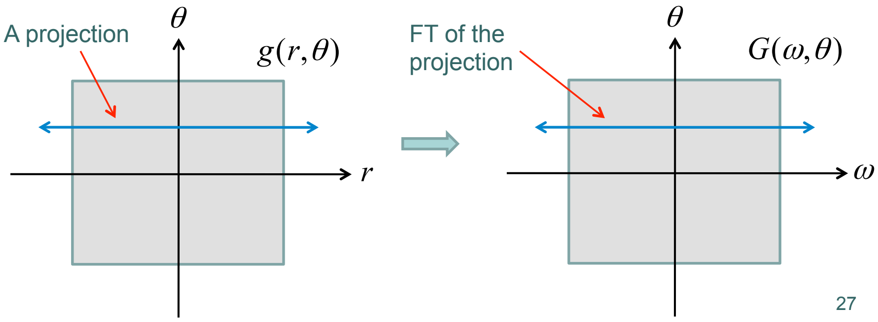
Fourier Transform of Projections

- Compute the 1-D Fourier transform of the Radon transform with respect to r.

$$G(\omega,\theta) = \int_{-\infty}^{\infty} g(r,\theta)\, e^{-j\omega r} dr$$



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Fourier Transform of Projections

$$G(\omega,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - r)\, e^{-j\omega r} dr\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \int_{-\infty}^{\infty} \delta(x\cos\theta + y\sin\theta - r)\, e^{-j\omega r} dr\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j\omega x\cos\theta} e^{-j\omega y\cos\theta} dx\, dy$$

$$= F(\omega\cos\theta, \omega\sin\theta)$$

A rotated line

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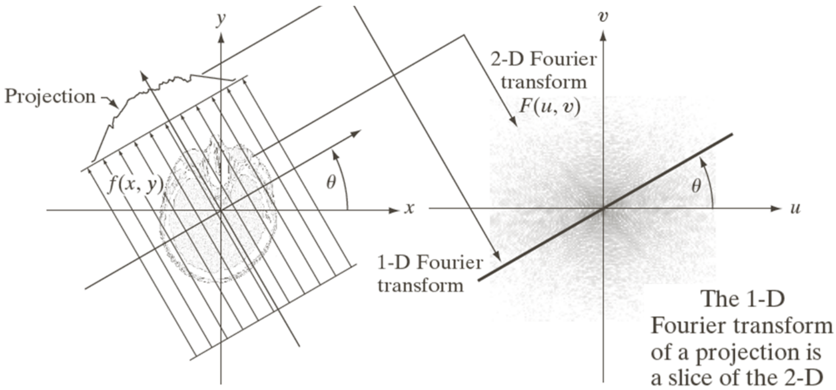
Fourier Slice (Projection Slice) Theorem



The 1-D Fourier transform of a projection is a slice of the 2-D Fourier transform

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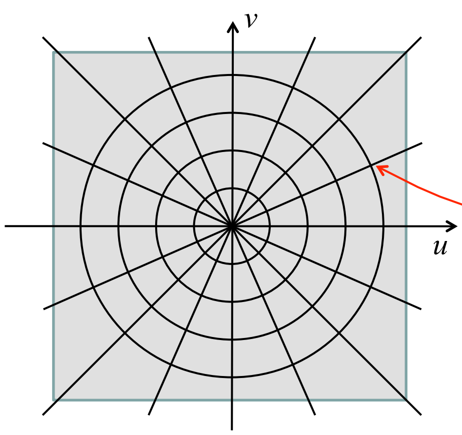
Polar Sampling

$F(u,v)$ at $u = \omega\cos\theta,\ v = \omega\sin\theta$



Uniform sampling along ω, θ

Polar sample locations for u, v

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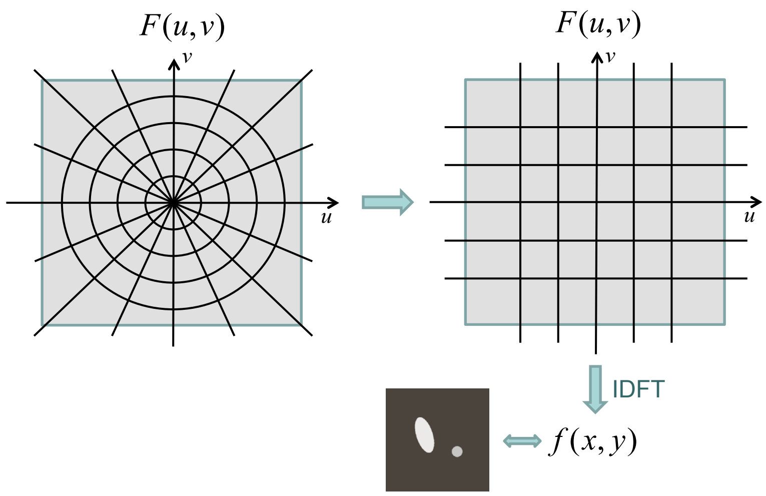
Polar to Rectangular Sampling Conversion



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Approach 2: Filtered Backprojections

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Reconstruction Equations

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(u,v)\,e^{jux}e^{jvy}du\,dv$$

$$=\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}F(\omega\cos\theta,\omega\sin\theta)\,e^{j\omega(x\cos\theta+y\sin\theta)}\omega\,d\omega\,d\theta$$

$$=\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}\omega\,G(\omega,\theta)\,e^{j\omega(x\cos\theta+y\sin\theta)}d\omega\,d\theta$$

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Correct Reconstruction from Backprojections

$$f(x,y)=\int_{-\pi/2}^{\pi/2}\left(\int_{-\infty}^{\infty}|\omega|\,G(\omega,\theta)\,e^{j\omega(x\cos\theta+y\sin\theta)}d\omega\right)d\theta$$

$$g(r,\theta)\;\Rightarrow\;G(\omega,\theta)\;\Rightarrow\;|\omega|\,G(\omega,\theta)\;\Rightarrow\;g'(r,\theta)$$

$$\Leftrightarrow\quad \hat{f}(x,y)=\int_{-\pi/2}^{\pi/2}g'(x\cos\theta+y\sin\theta,\theta)\,d\theta$$

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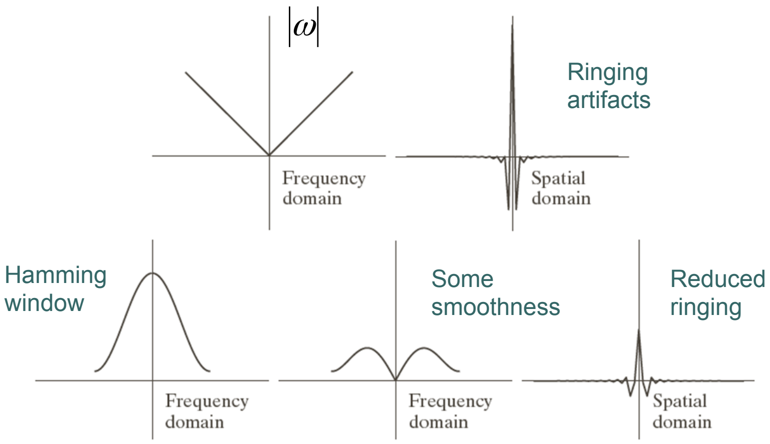
Ramp Filter



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Reconstruction from Partial Projections

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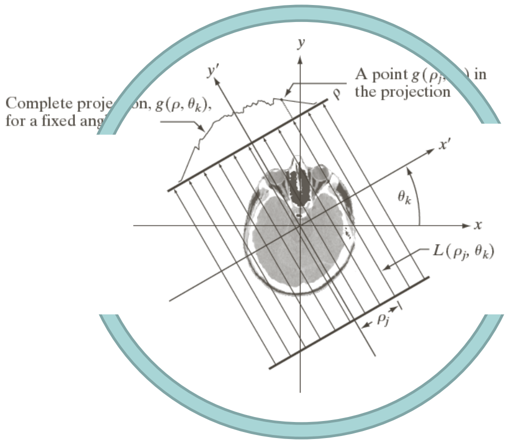
Missing Projections



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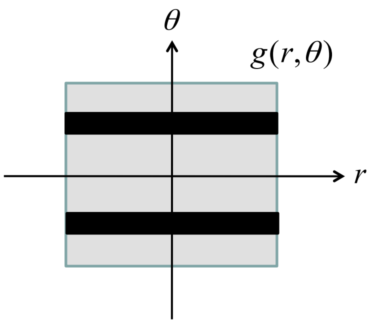
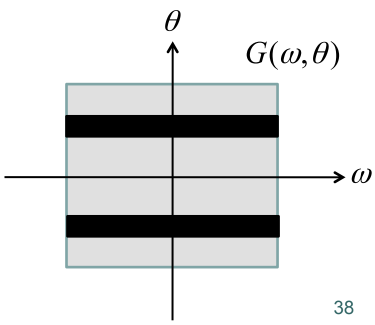
Missing Angles in the Radon Transform

$$G(\omega,\theta) = \int_{-\infty}^{\infty} g(r,\theta)\, e^{-j\omega r}\, dr$$





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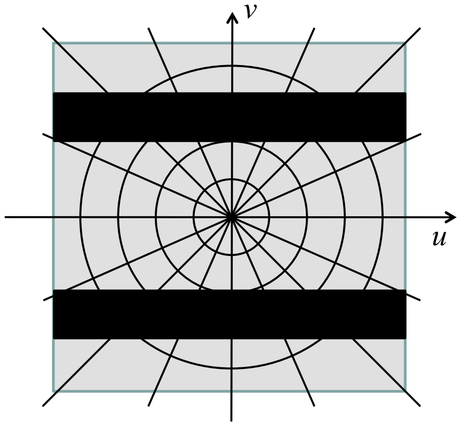
Missing Samples in the 2D Fourier Transform

$F(u,v)$ at $u = \omega\cos\theta,\ v = \omega\sin\theta$



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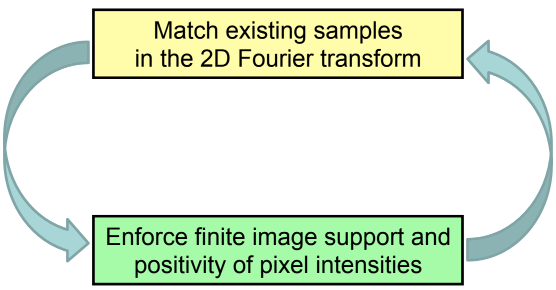
POCS Solution



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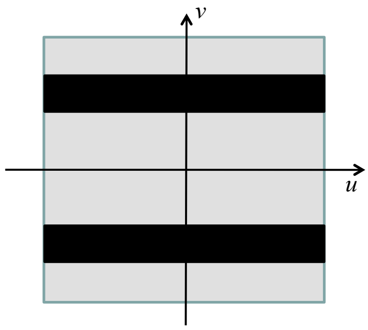
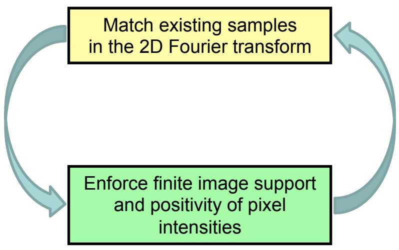
Project 3.3a

Image Reconstruction

Due 12.01.2013 Sunday

Image Reconstruction via POCS





Problem 3.3a

- Select a 256x256 monochrome image and display it.
- Pad zeros to the right and to the bottom of this image and then take its 512x512 DFT—call it $D(k,l)$.
- Define $D'(k,l)$ to be the observed DFT of the image with missing segments:

$$D'(k,l) = \begin{cases} 0 & 50 \le k \le 150 \\ 0 & 362 \le k \le 462 \\ D(k,l) & elsewhere \end{cases}$$

- Take the inverse DFT of $D'(k,l)$ and apply the "256x256 support" and "0—255 amplitude" constraint.
- Take the DFT of the image obtained in the previous step and match the observed samples of $D'(k,l)$.
- Repeat the previous two steps 5 times and display the results after each iteration. Comment on the results.

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Next Lecture

- IMAGE COMPRESSION

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